M/EEG source analysis

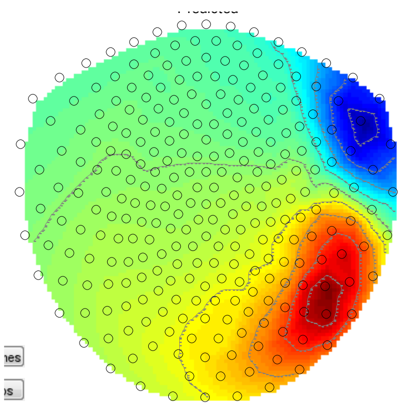
José David López

Key points:

- What is an ill-posed inverse problem
- Prior knowledge- links to popular algorithms.
- Validation of prior knowledge/ Model evidence

The forward problem

Prediction



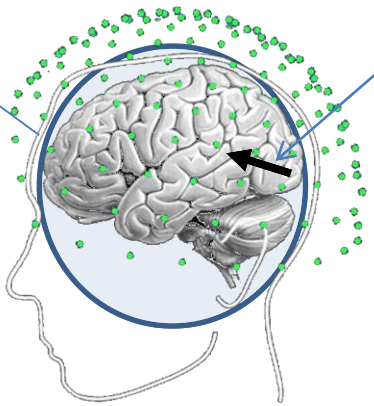
M/EEG
sensors

Lead field (L) is the sensitivity of the M/EEG system to a dipolar source at a particular location

Lead fields (L)

Head Model

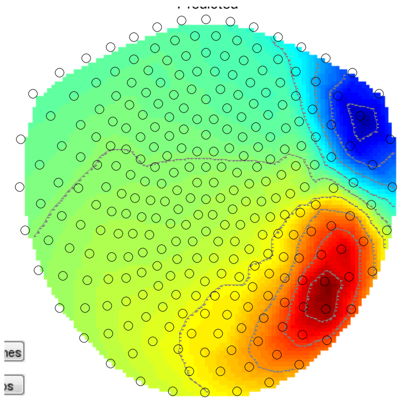
Dipolar source



Model describes
conductivity & geometry

Analogy
2+3= ?

The Inverse problem



M/EEG
sensors

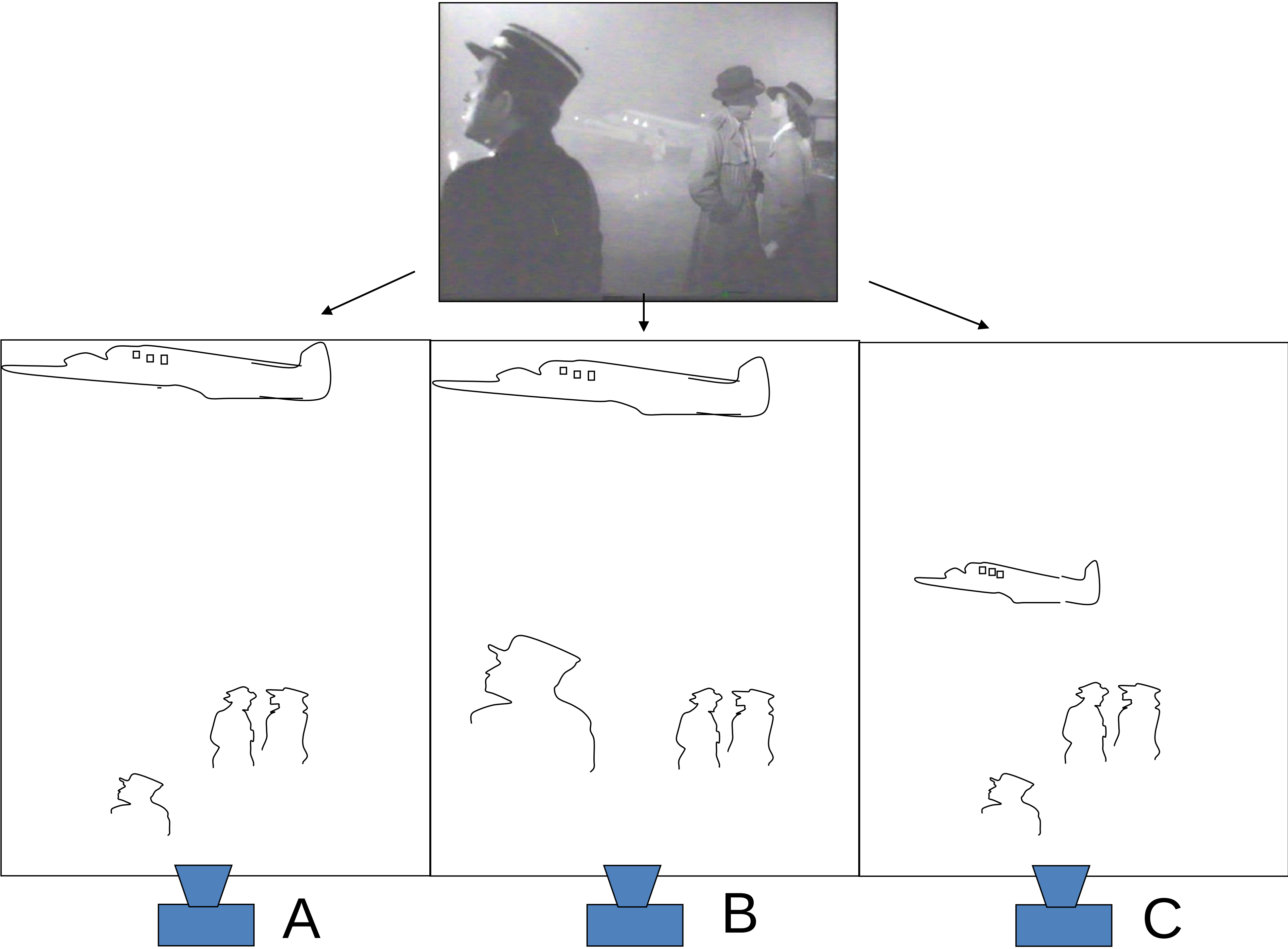
Measurement

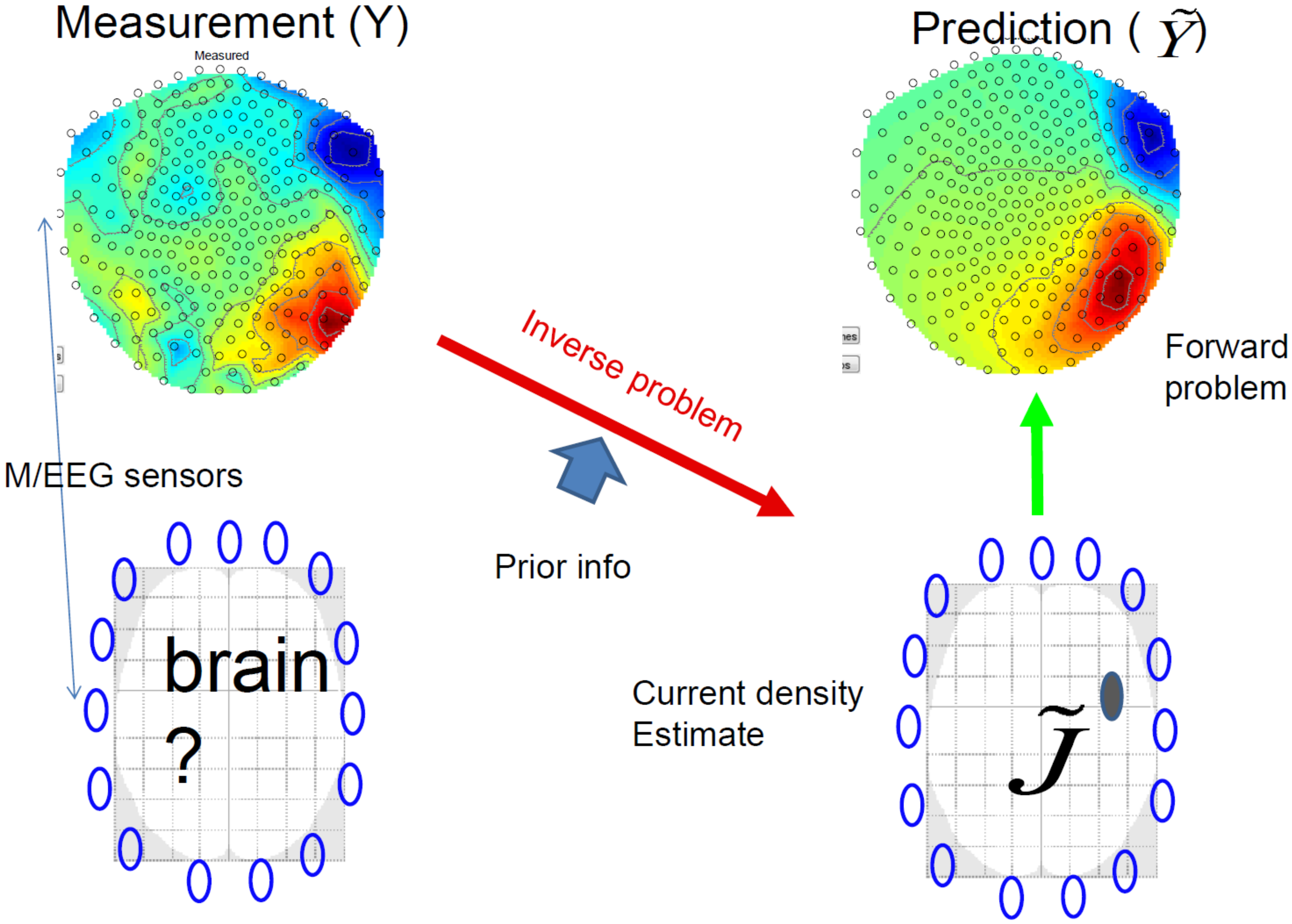
Which brain sources gave rise to these measured data ?

Inference

Analogy
5= ? +?

Inverse problems aren’t difficult


A
B
C



Measurement (Y)

Prediction ($\tilde{Y}$)

Forward problem

Inverse problem

M/EEG sensors

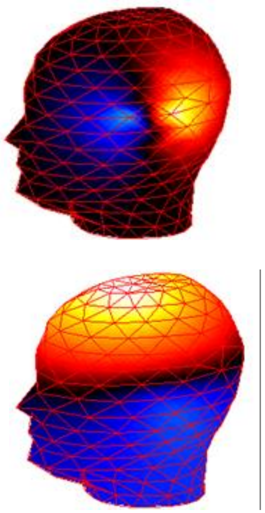
Prior info

brain ?

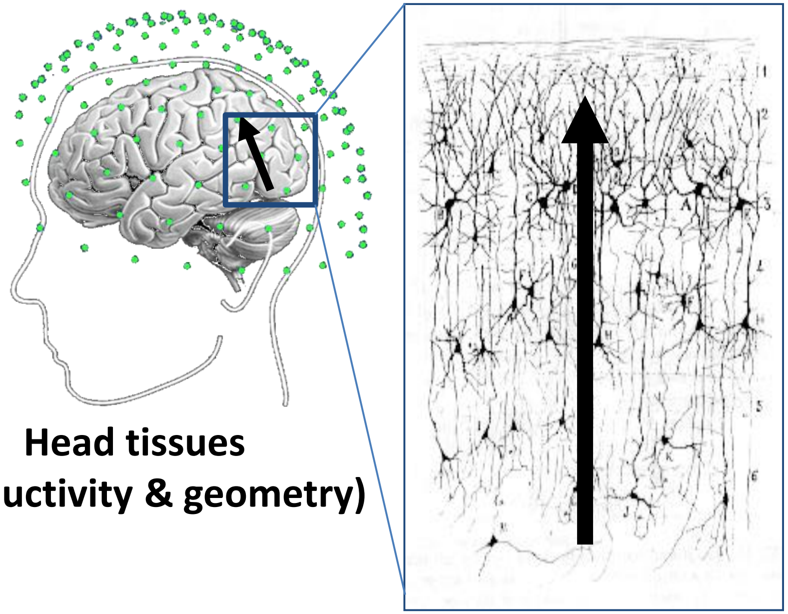
Current density Estimate

$\tilde{J}$

The forward problem



Lead fields

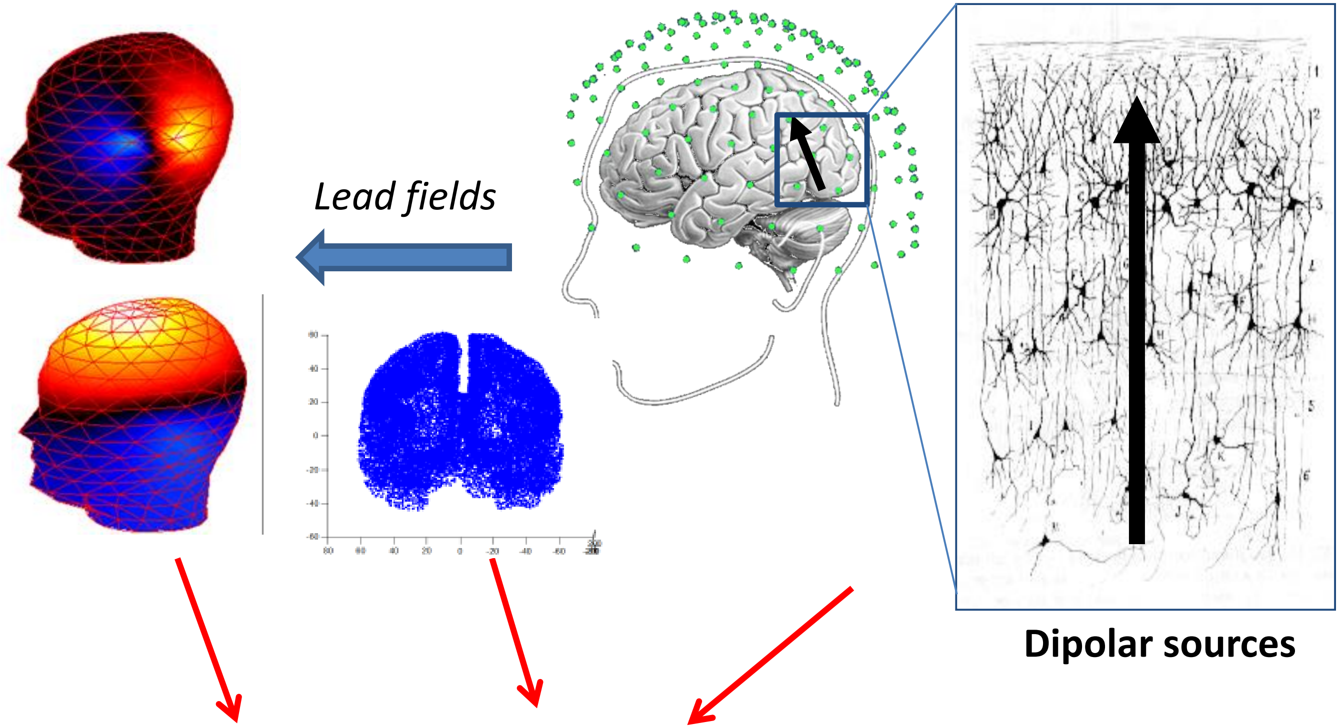


Head tissues (conductivity & geometry)

Neurons


[redacted]

The forward problem



MEG

EEG

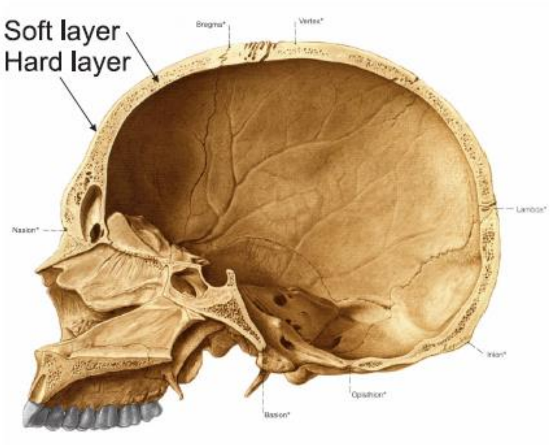
Lead fields

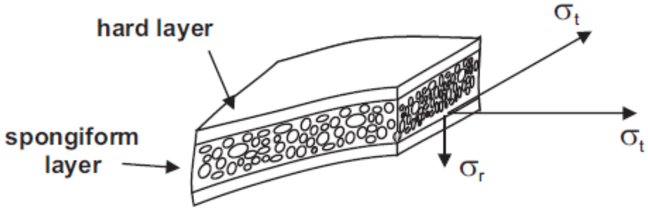
Dipolar sources

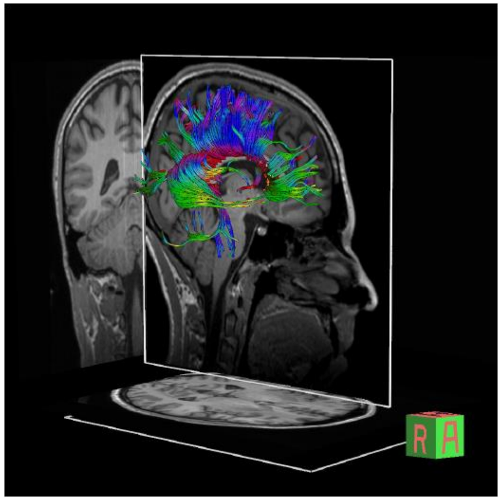
$$Y = LJ + \epsilon$$

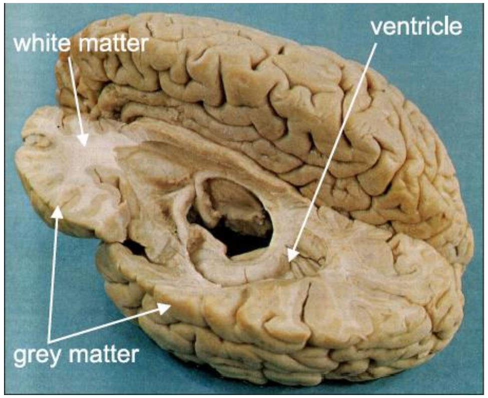
[redacted]

Head model (gain matrix)





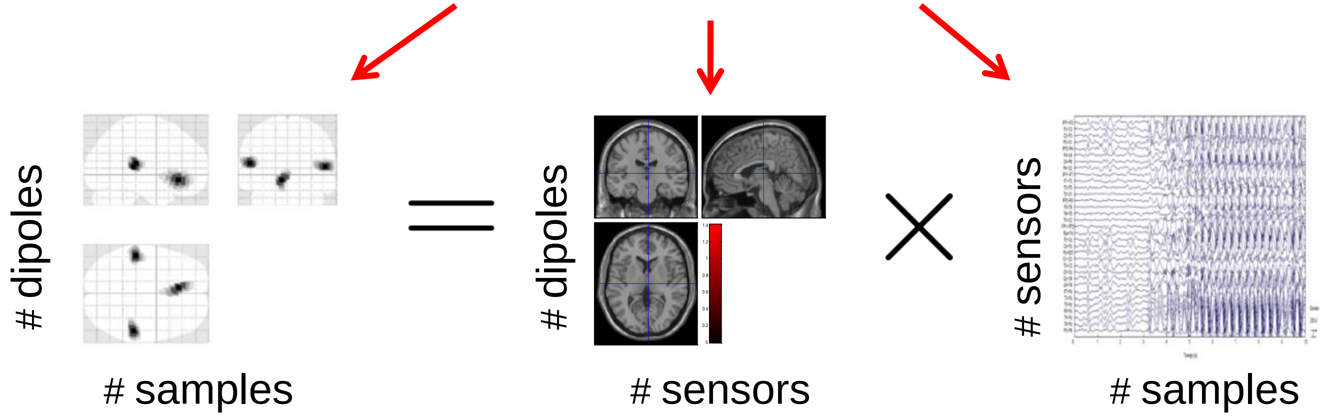




MEG/EEG brain imaging

With the acquired data we may recover the neural activity

$$J = L^{-1}Y$$



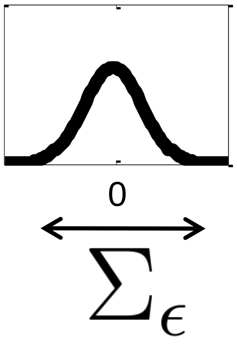
But the problem is ill-posed:

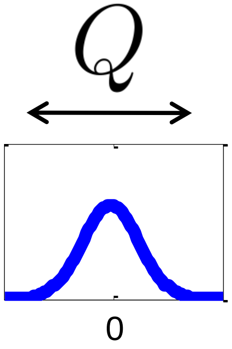
NON INVERTIBLE!!! → Infinite solutions!!!

Bayesian formulation

We must include prior information:

$$Y = LJ + \epsilon \longrightarrow$$





then we can use the Bayes' theorem:

Adjusted with the data

Assumed

$$p(J|Y) = \frac{p(Y|J)p(J)}{p(Y)}$$

Constant

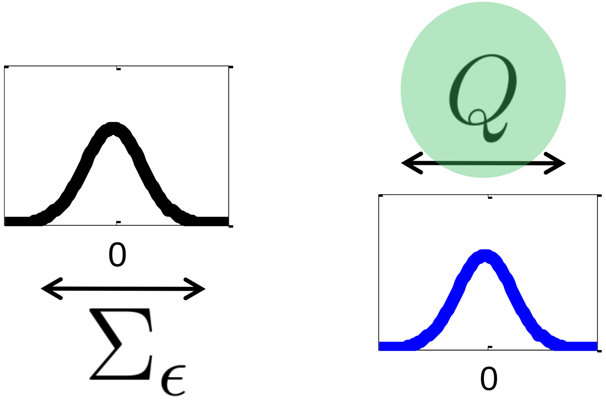
and solving for Gaussian assumptions:

$$\widehat{J} = E[p(J|Y)] \longrightarrow \boxed{\widehat{J} = QL^T(\Sigma_\epsilon + LQL^T)^{-1}Y}$$

Bayesian formulation

We must include prior information:

$$Y = LJ + \epsilon \longrightarrow$$



then we can use the Bayes' theorem:

Adjusted with the data

Assumed

$$p(J|Y) = \frac{p(Y|J)p(J)}{p(Y)}$$

Constant

and solving for Gaussian assumptions:

$$\hat{J} = E[p(J|Y)] \longrightarrow \boxed{\hat{J} = QL^T(\Sigma_\epsilon + LQL^T)^{-1}Y}$$

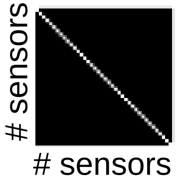
Grech et al., 2008

Prior covariance matrices

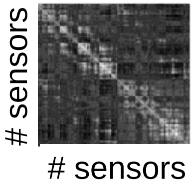
PRIOR NOISE COVARIANCE

Independent sensor noise

$$\Sigma_\epsilon = h_0 I$$

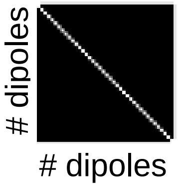


Empty room activity



PRIOR COVARIANCE OF SOURCE SPACE ACTIVITY

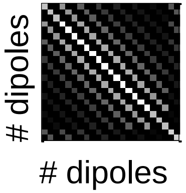
Minimum norm $Q = I$ →



Non informative

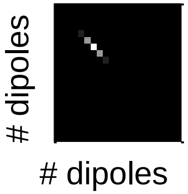
$$\widehat{J} = L(\Sigma_\epsilon + LL^T)^{-1}Y$$

LORETA-like: $Q = e^{\sigma G_L}$ →



Smoothed

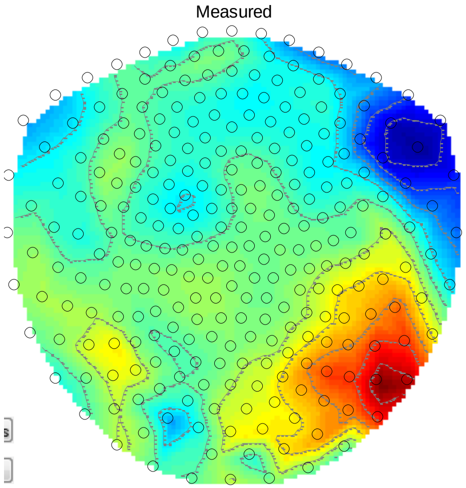
Beamformers: $Q = (L^T(YY^T)^{-1}L)^{-1}$ →



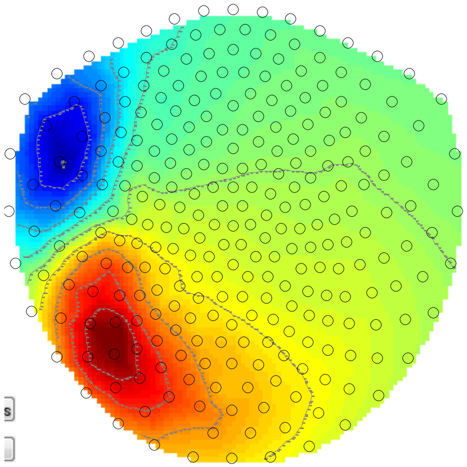
Data based

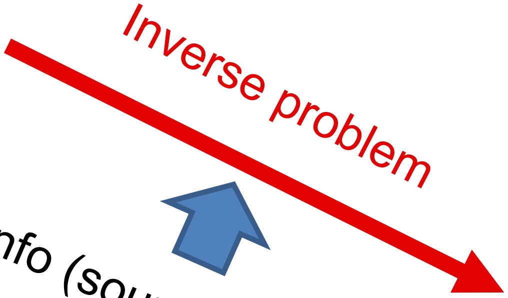
Illustrative example

Y (measured field)



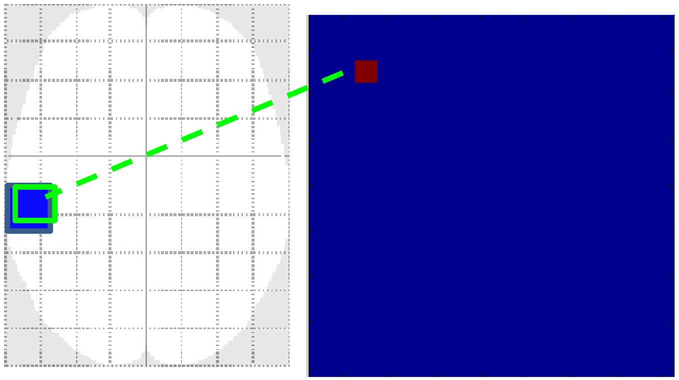
PREDICTED



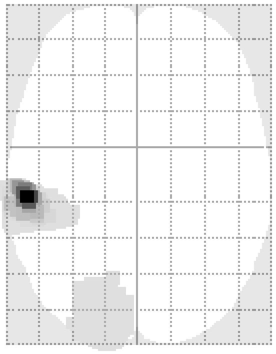


Inverse problem

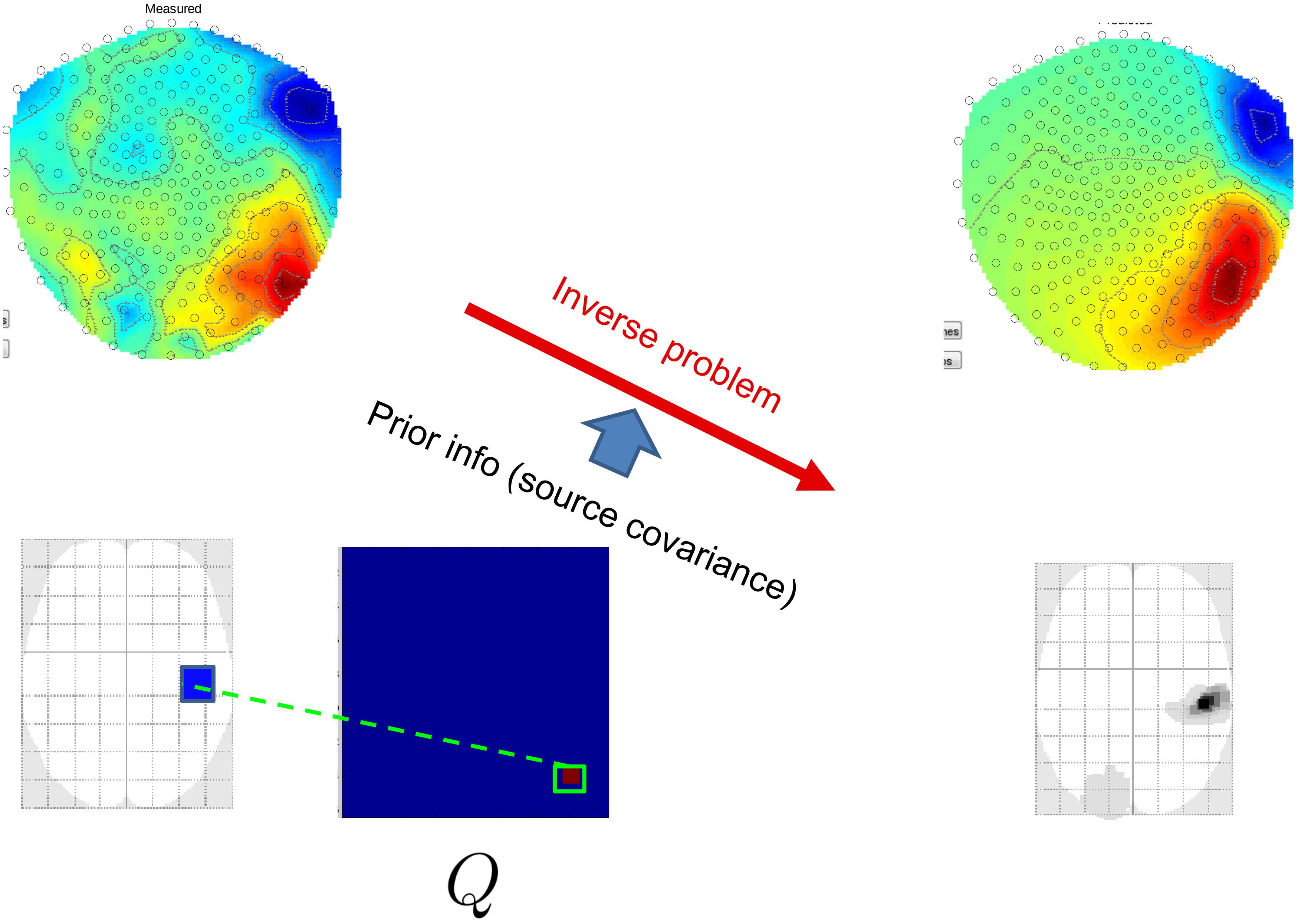
Prior info (source covariance)



Q



Single dipole fit



Y (measured field)

PREDICTED

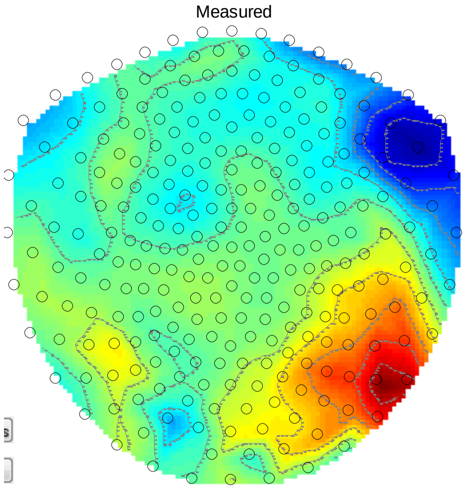
Inverse problem

Prior info (source covariance)

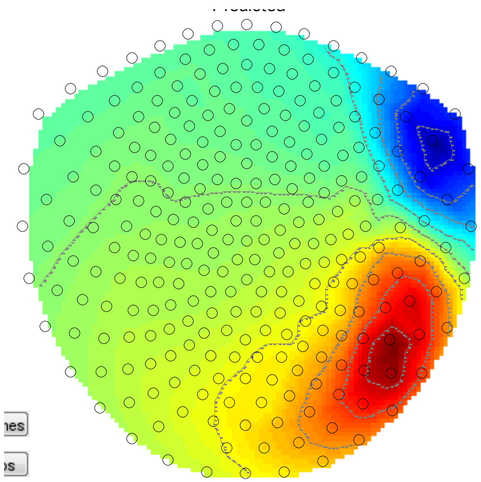
Q

Two dipole fit

Y (measured field)

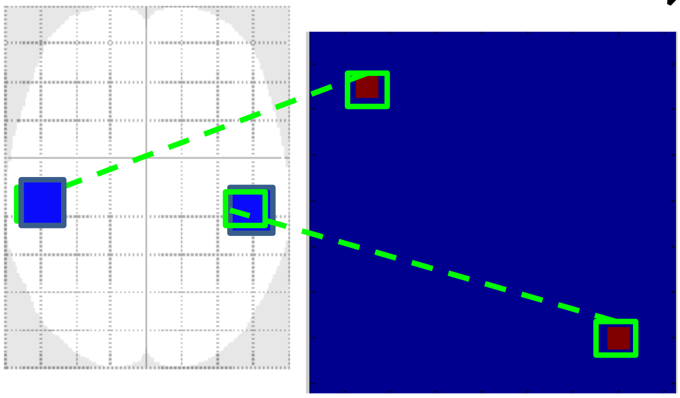


PREDICTED

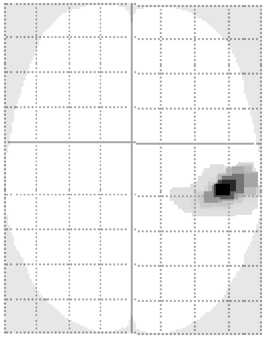


Inverse problem

Prior info (source covariance)

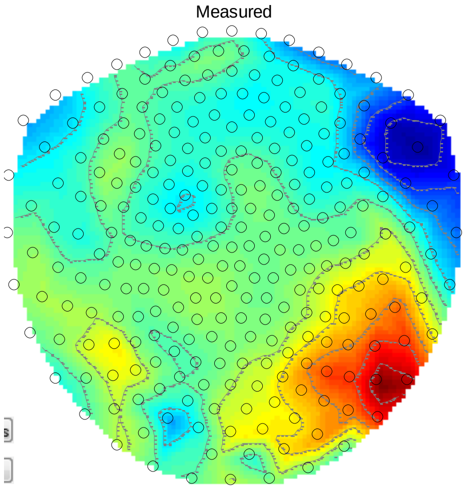


Q

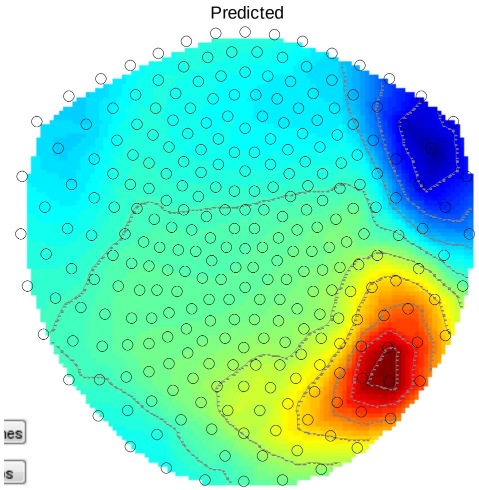


Minimum norm

Y (measured field)

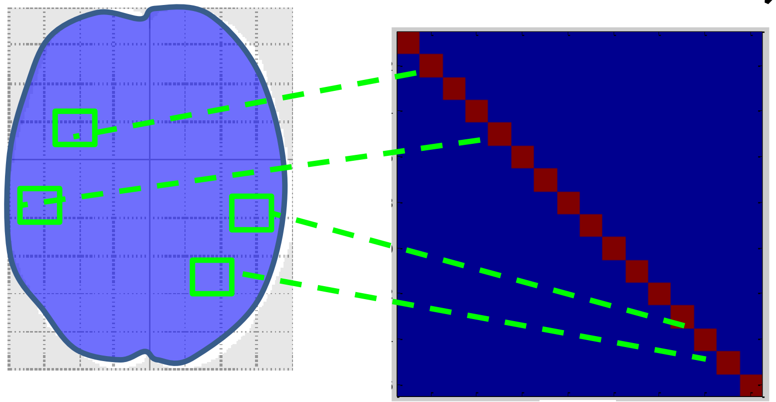


PREDICTED

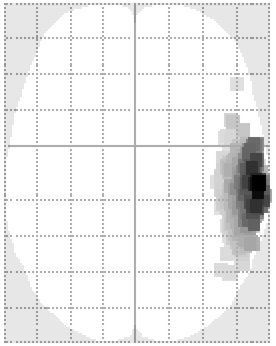


Inverse problem

Prior info (source covariance)

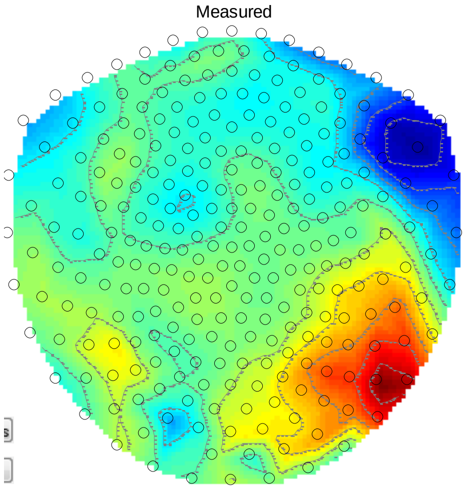


Q

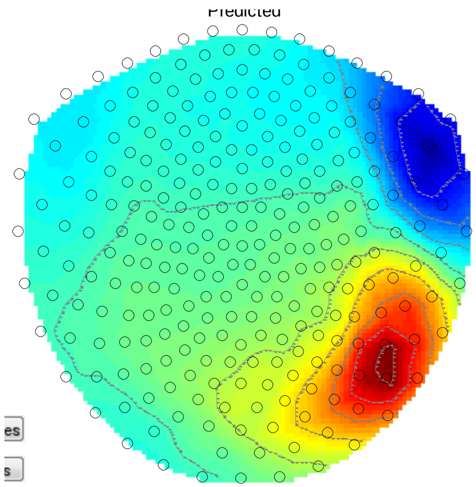


Beamformer (adaptive algorithm/ Empirical)

Y (measured field)



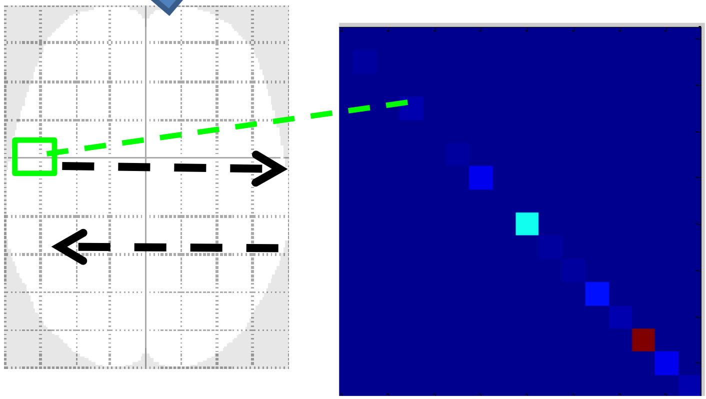
PREDICTED



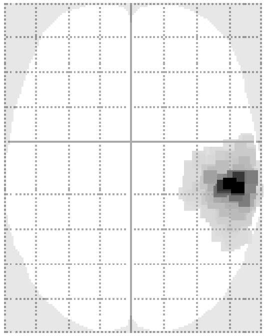
Inverse problem

Prior info (source covariance)

Projection onto lead field*



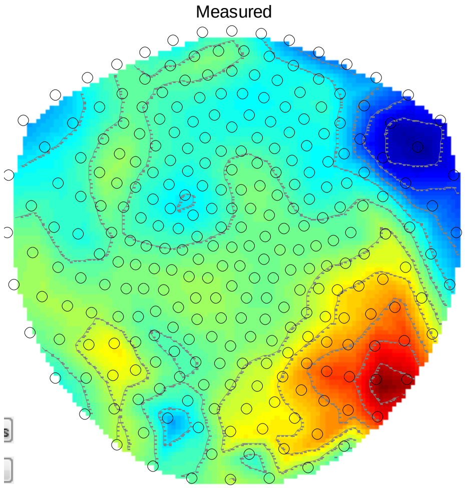
Q



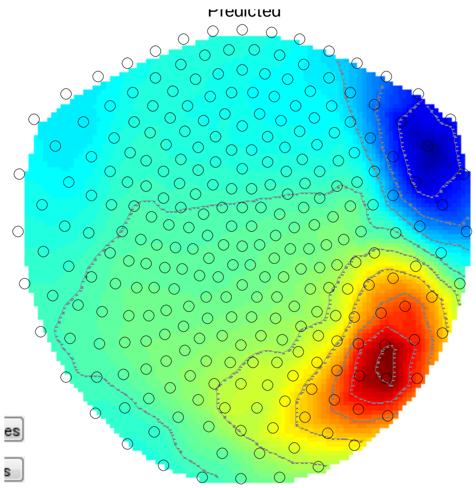
*Assuming no correlated sources

fMRI biased dSPM
(Dale et al. 2000)

Y (measured field)

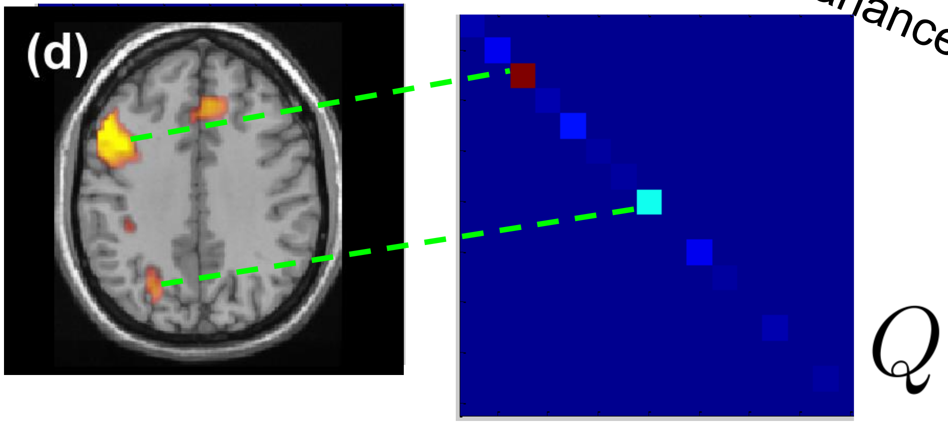


PREDICTED

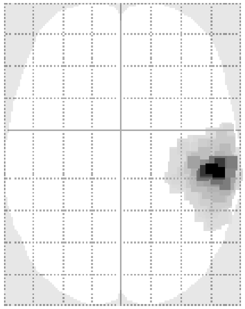


Inverse problem

Prior info (source covariance)

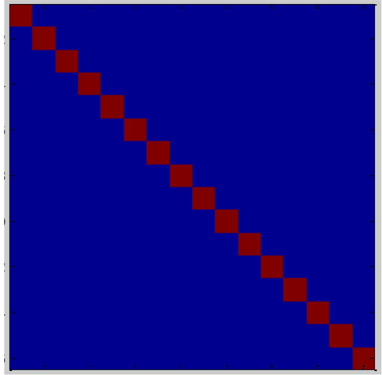


fMRI data

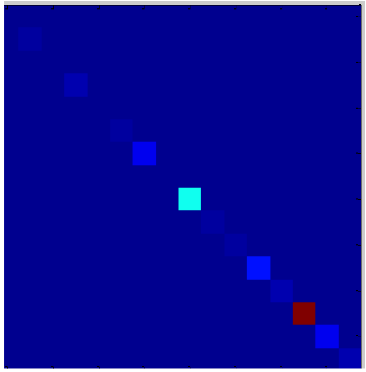


Maybe…

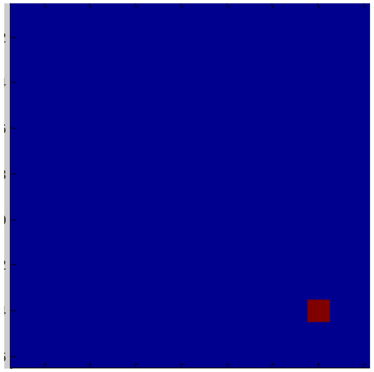
Summary: Some popular priors



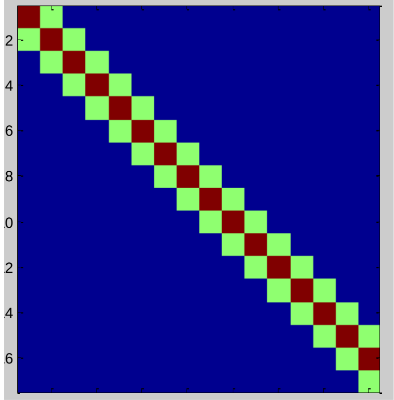
Minimum norm



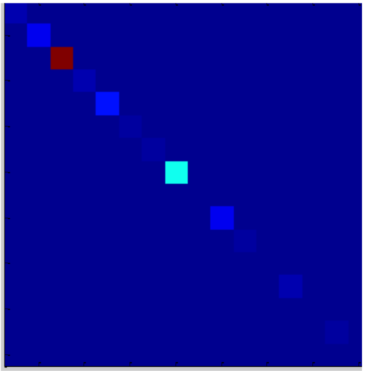
SAM,DICs
Beamformer



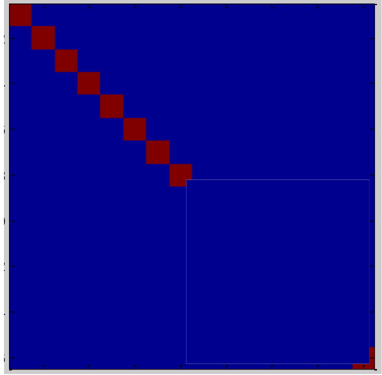
Dipole fit



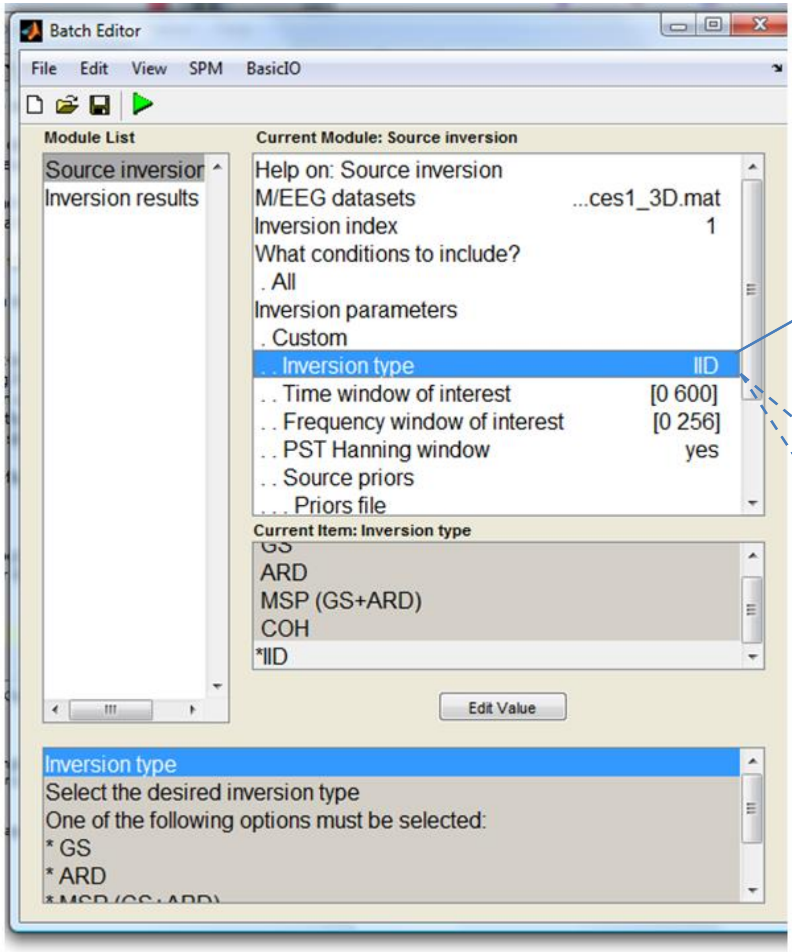
LORETA



fMRI biased
dSPM



?



Source inversion
Inversion results

Current Module: Source inversion

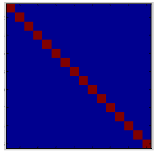
Help on: Source inversion
M/EEG datasets ...ces1_3D.mat
Inversion index 1
What conditions to include?
. All
Inversion parameters
. Custom
. . Inversion type IID
. . Time window of interest [0 600]
. . Frequency window of interest [0 256]
. . PST Hanning window yes
. . Source priors
. . . Priors file

Current Item: Inversion type

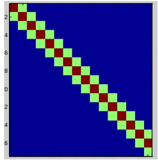
GS
ARD
MSP (GS+ARD)
COH
*IID

Inversion type
Select the desired inversion type
One of the following options must be selected:
* GS
* ARD

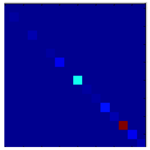
Minimum Norm (IID - independent and identically distributed)



LORETA (COH- coherent)



Empirical Bayes Beamformer (EBB)



Multiple Sparse Priors (MSP/ Greedy Search (GS) Automatic relevance determination (ARD))

Summary

- MEG inverse problem requires prior information in the form of a source covariance matrix.
- Different inversion algorithms- SAM, DICS, LORETA, Minimum Norm, dSPM… just have different prior source covariance structure.
- Historically- different MEG groups have tended to use different algorithms/acronyms.

See
Mosher et al. 2003, Friston et al. 2008, Wipf and Nagarajan 2009, Lopez et al. 2014

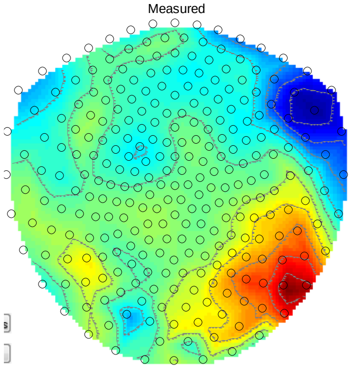
Software

- ***SPM12:*** ***http://www.fil.ion.ucl.ac.uk/spm/software/spm12/***
- ***DAiSS-*** SPM12 toolbox for Data Analysis in Source Space (beamforming, minimum norm and related methods), developed by Vladimir Litvak:

  https://github.com/spm/DAiSS

- **Fieldtrip** : http://fieldtrip.fcdonders.nl/
- **Brainstorm**: http://neuroimage.usc.edu/brainstorm/
- **MNE**: http://martinos.org/mne/stable/index.html

Y (measured field)



How do we chose between priors ?

Are you drunk?

You won the lottery

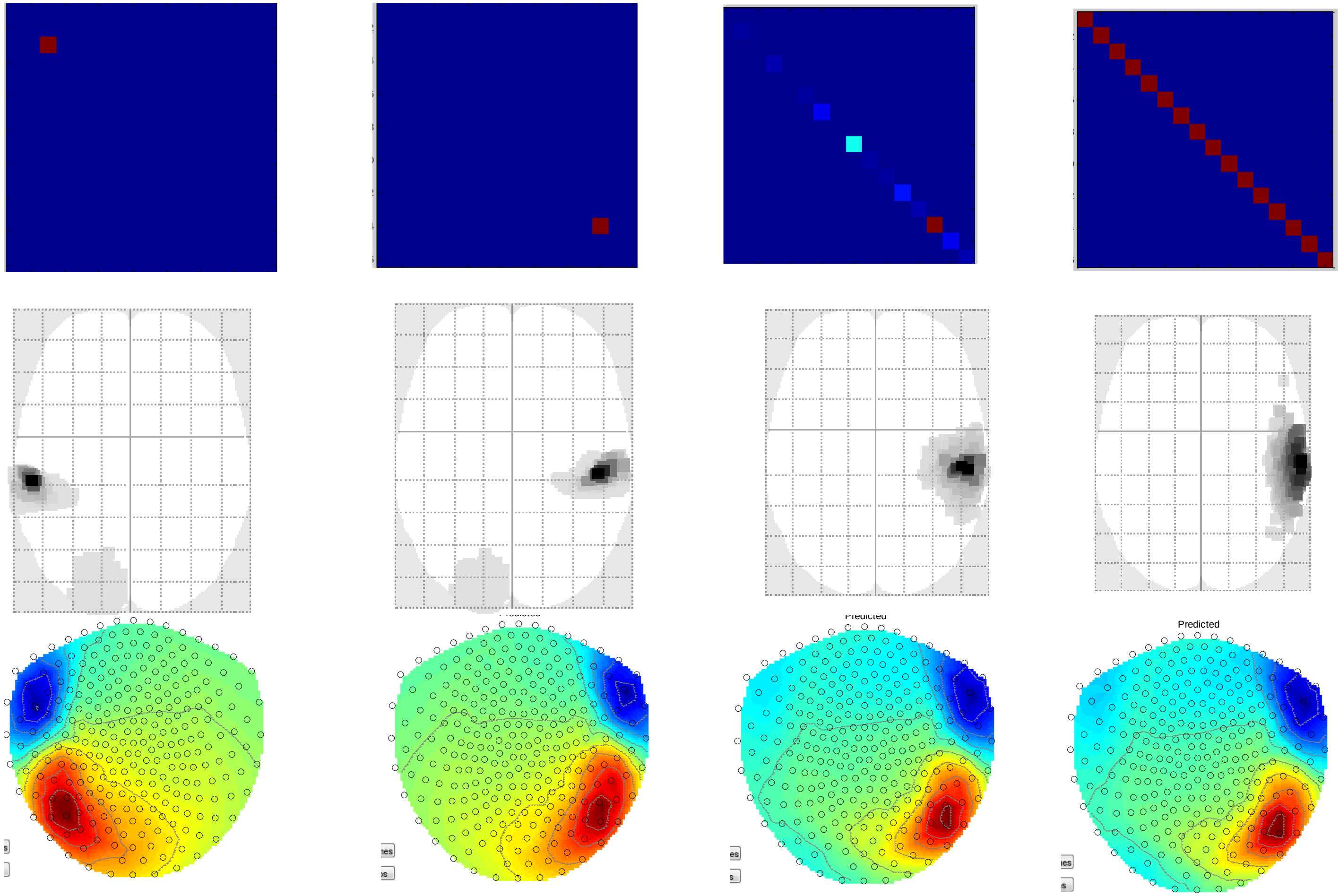
Beamformer

Minimum norm

Prior

Estimated Current flow

Predicted data



Variance explained

11 %

96%

97%

98%

Penny, 2012

Negative variational free energy (1)

$$\boxed{\log p(\widehat{Y}) = F + KL[q(h)||p(h|Y)]}$$

the divergence will be zero if the approximated distribution is equal to the posterior one:

$$q(h) = p(h|Y) \Longrightarrow F = \log p(Y)$$

Penny, 2012

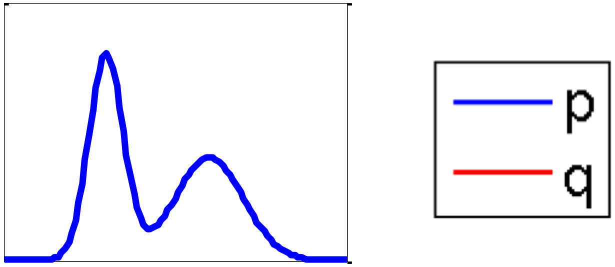
Negative variational free energy (1)

$$\boxed{\log p(\widehat{Y}) = F + KL[q(h)||p(h|Y)]}$$

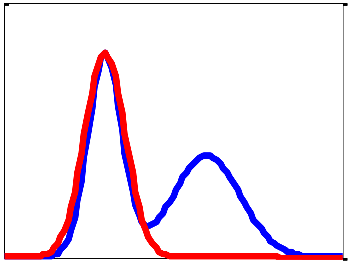
the divergence will be zero if the approximated distribution is equal to the posterior one:

$$q(h) = p(h|Y) \Longrightarrow F = \log p(Y)$$

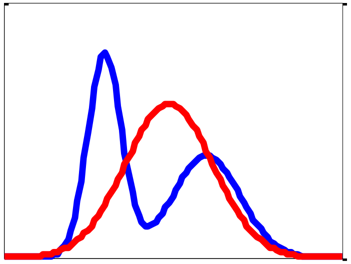
$$q_0(h) = \mathcal{N}(h; \nu, \Pi^{-1}) \Longrightarrow q(h) = \mathcal{N}(h; \widehat{h}, \Sigma_h)$$



$p(h|Y)$



Wrong



BETTER!

Friston et al., 2007

Negative variational free energy (2)

The free energy can be expressed as:

$$\boxed{\begin{aligned} F = &-\frac{N_t}{2}\mathrm{tr}(C_Y\Sigma_Y^{-1}) - \frac{N_t}{2}\log|\Sigma_Y| - \frac{N_cN_t}{2}\log(2\pi) \\ &-\frac{1}{2}\mathrm{tr}\left((\widehat{h}-\nu)^T\Pi(\widehat{h}-\nu)\right) + \frac{1}{2}\log|\Pi\Sigma_h| \end{aligned}}$$

Reducing constant terms and assuming zero mean priors:

$$F = -\mathrm{trace}\left(\frac{YY^T}{\Sigma_\epsilon + LQL^T}\right) - \log|\Sigma_\epsilon + LQL^T|$$

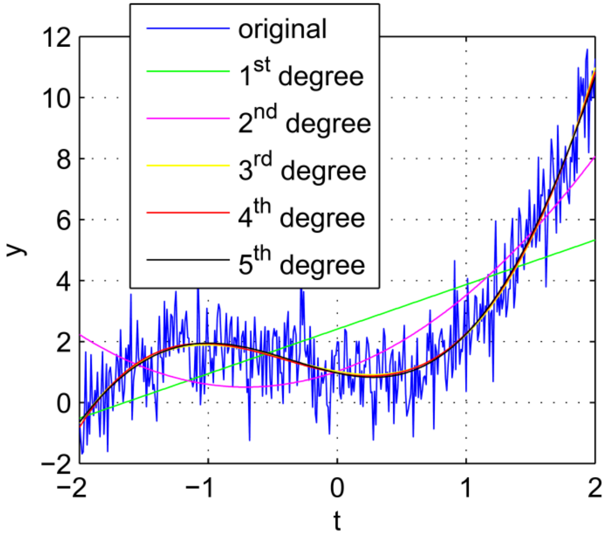
→ **Accuracy**

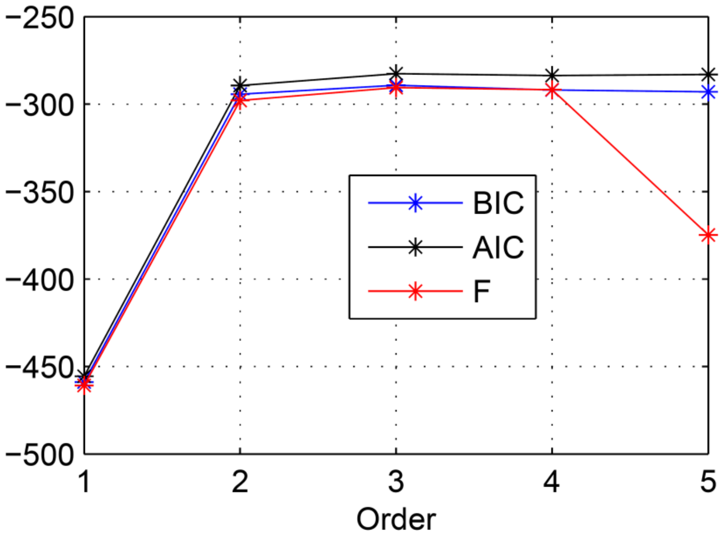
$$-\mathrm{trace}(h^T\Pi h) + \log|\Pi\Sigma_h|$$

→ **Complexity**

Trade-off between accuracy and complexity

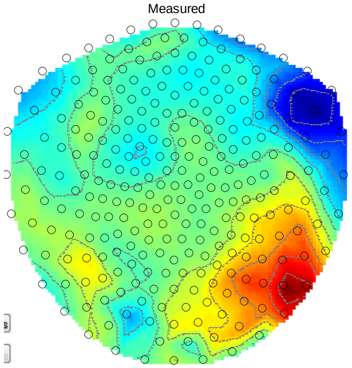
Approach	**Complexity term**
AIC (Akaike, 1974)	N_q
BIC (Schwarz, 1978)	$\frac{N_q}{2} \log N_t$
Linear function (Wipf and Nagarajan, 2009)	h
free energy (Friston et al., 2008)	$\frac{1}{2}\text{tr}\left((h-\nu)^T \Pi (h-\nu)\right) - \frac{1}{2}\log\lvert\Pi\Sigma_h\rvert$





How do we chose between priors ?

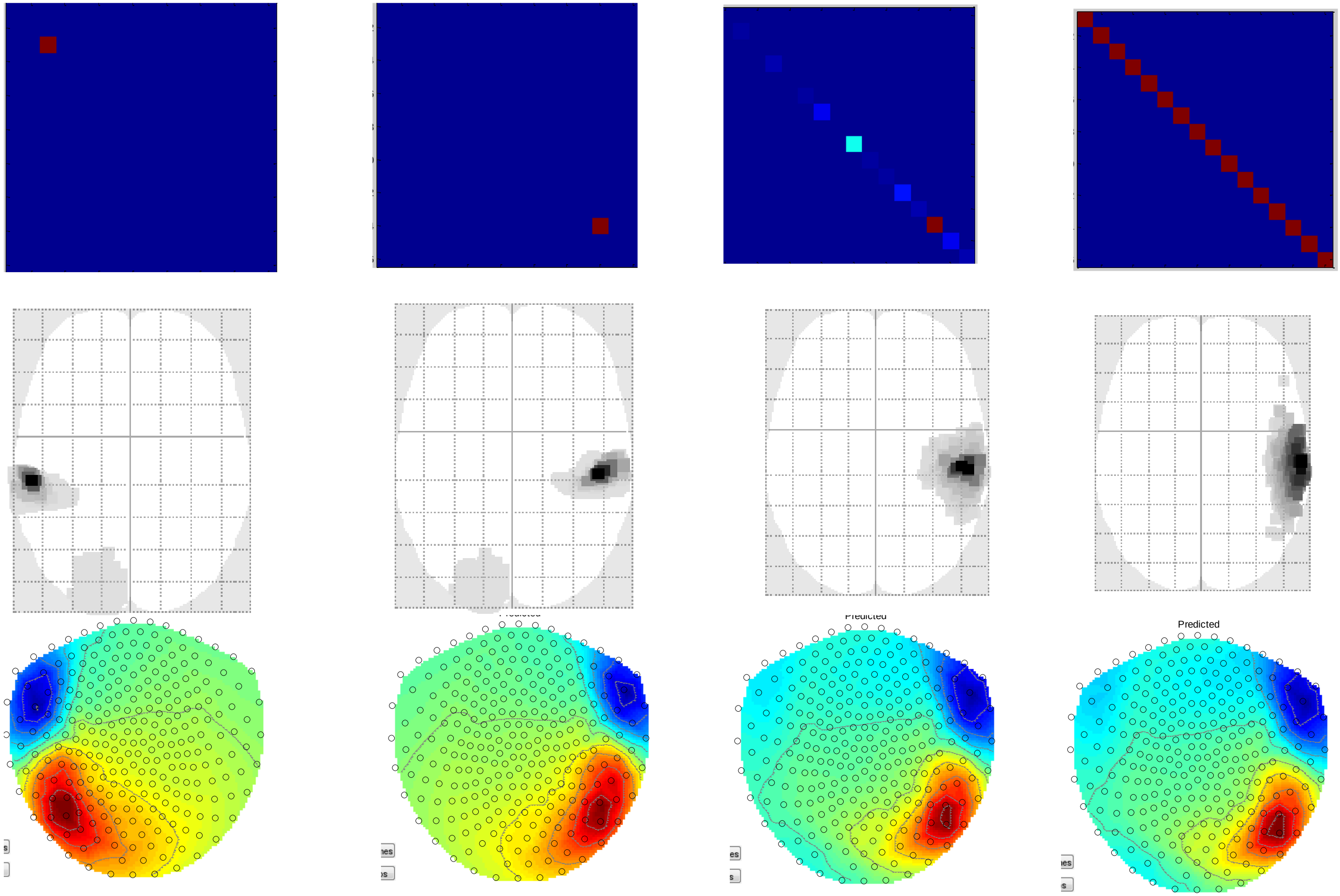
Y (measured field)



Prior

Estimated Current flow

Predicted data



	Are you drunk?	You won the lottery	Beamformer	Minimum norm
Variance explained	11 %	96%	97%	98%

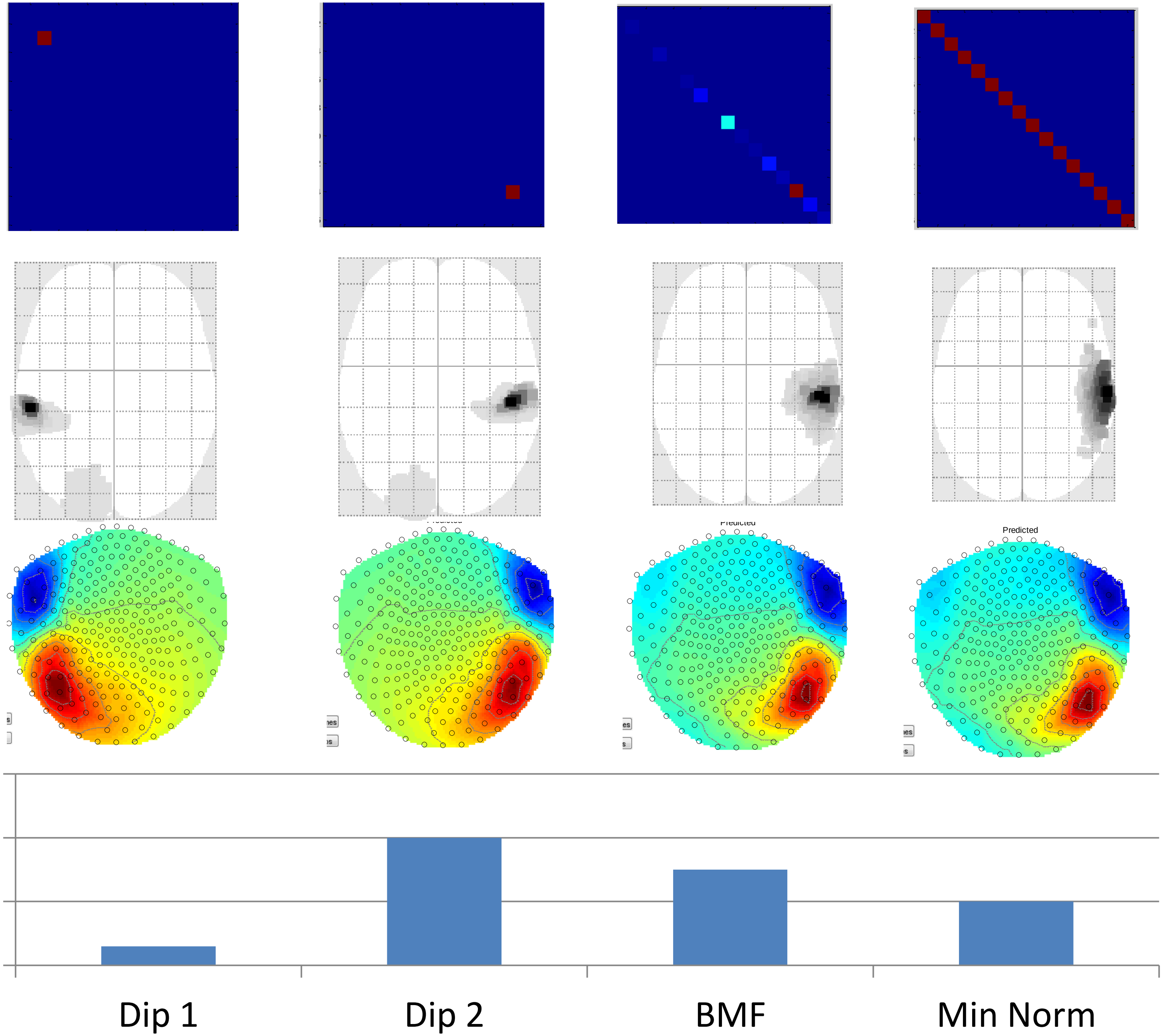
How do we chose between priors ?

Prior

Estimated Current flow

Predicted data

Free energy (log model evidence)



Friston et al., 2008

Multiple sparse priors (1)

All prior information can be included as the linear combination of a set of covariance components

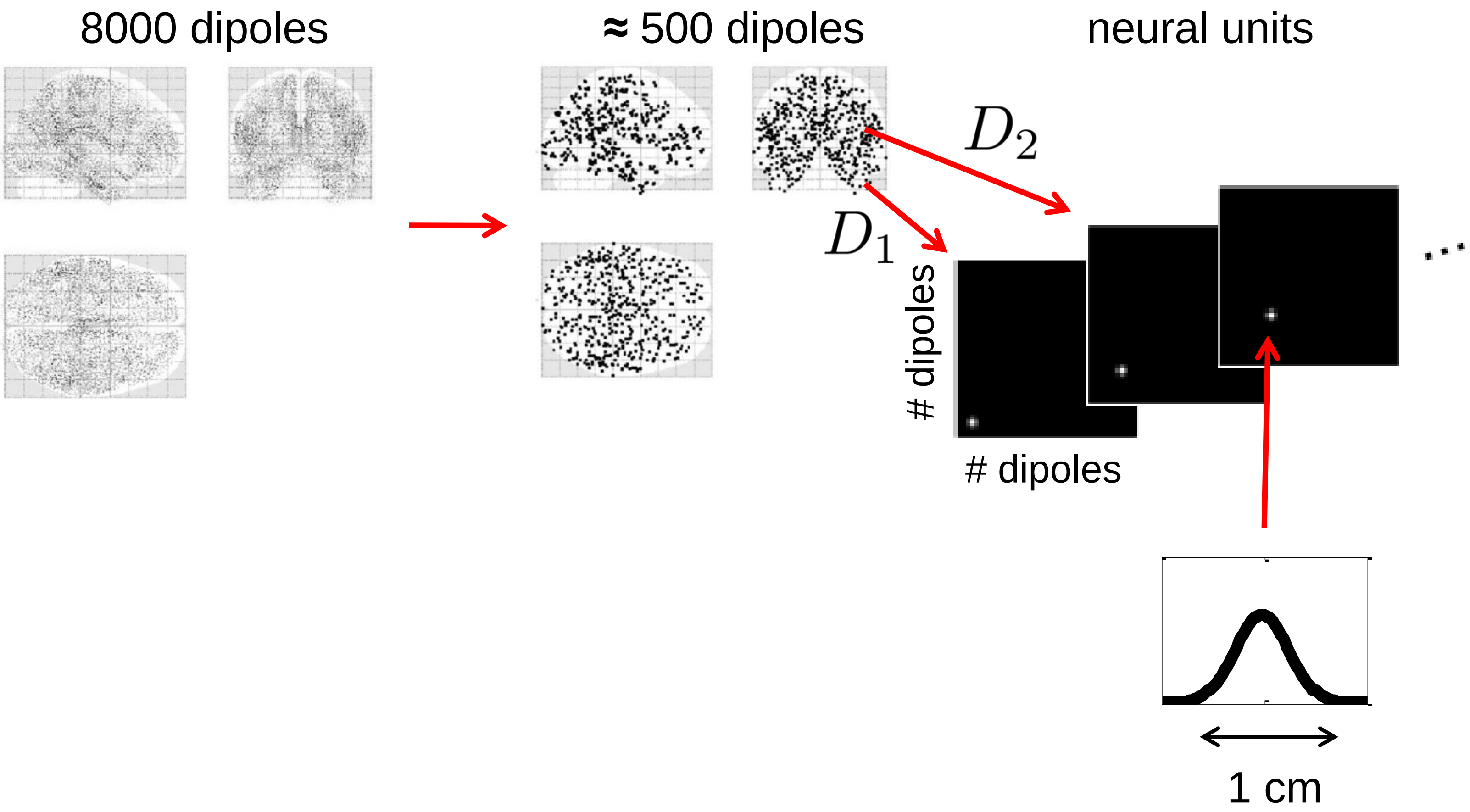
$$\widehat{J} = QL^T(\Sigma_\epsilon + LQL^T)^{-1}Y$$

$$\boxed{Q = \sum_{i=1}^{N_q} h_i D_i}$$

$$D = \{D_1, \ldots, D_{N_q}\}$$

$$h = \{h_1, \ldots, h_{N_q}\}$$

Multiple sparse priors (2)

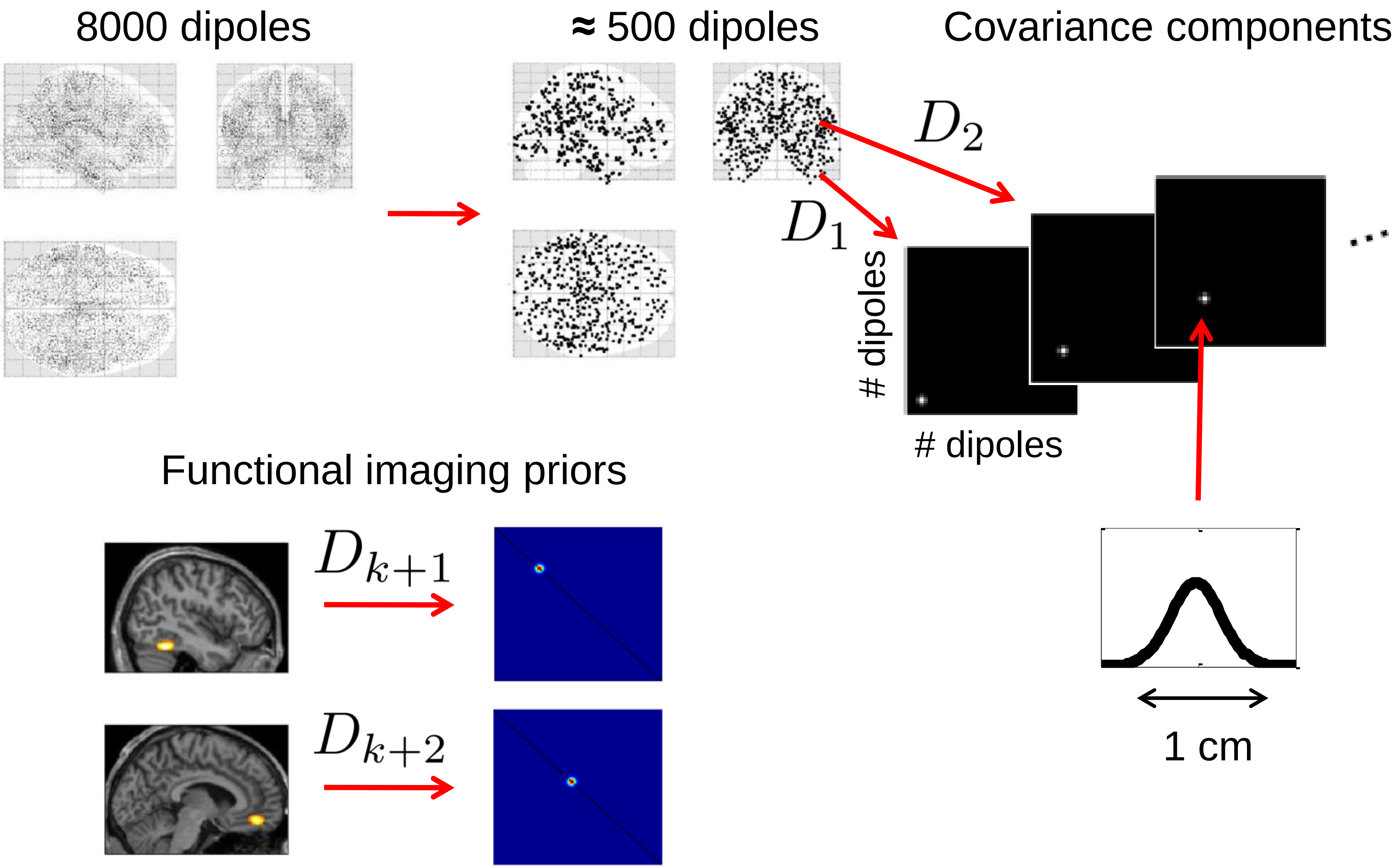


8000 dipoles

≈ 500 dipoles

neural units

Multiple sparse priors (2)



8000 dipoles

≈ 500 dipoles

Covariance components

D_2

D_1

dipoles

dipoles

...

Functional imaging priors

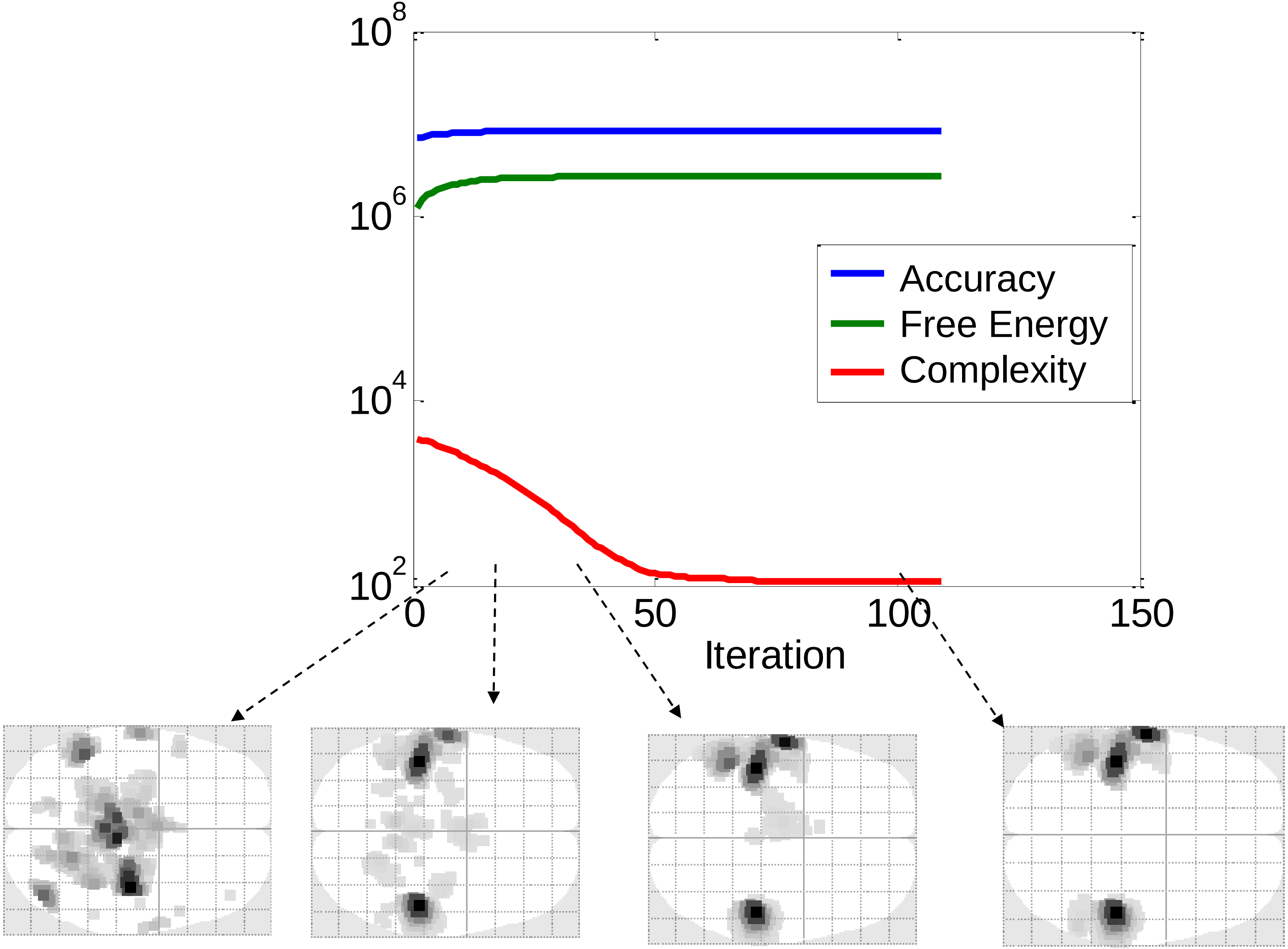
D_{k+1}

D_{k+2}

1 cm

Multiple Sparse priors

So now construct the priors to maximise model evidence



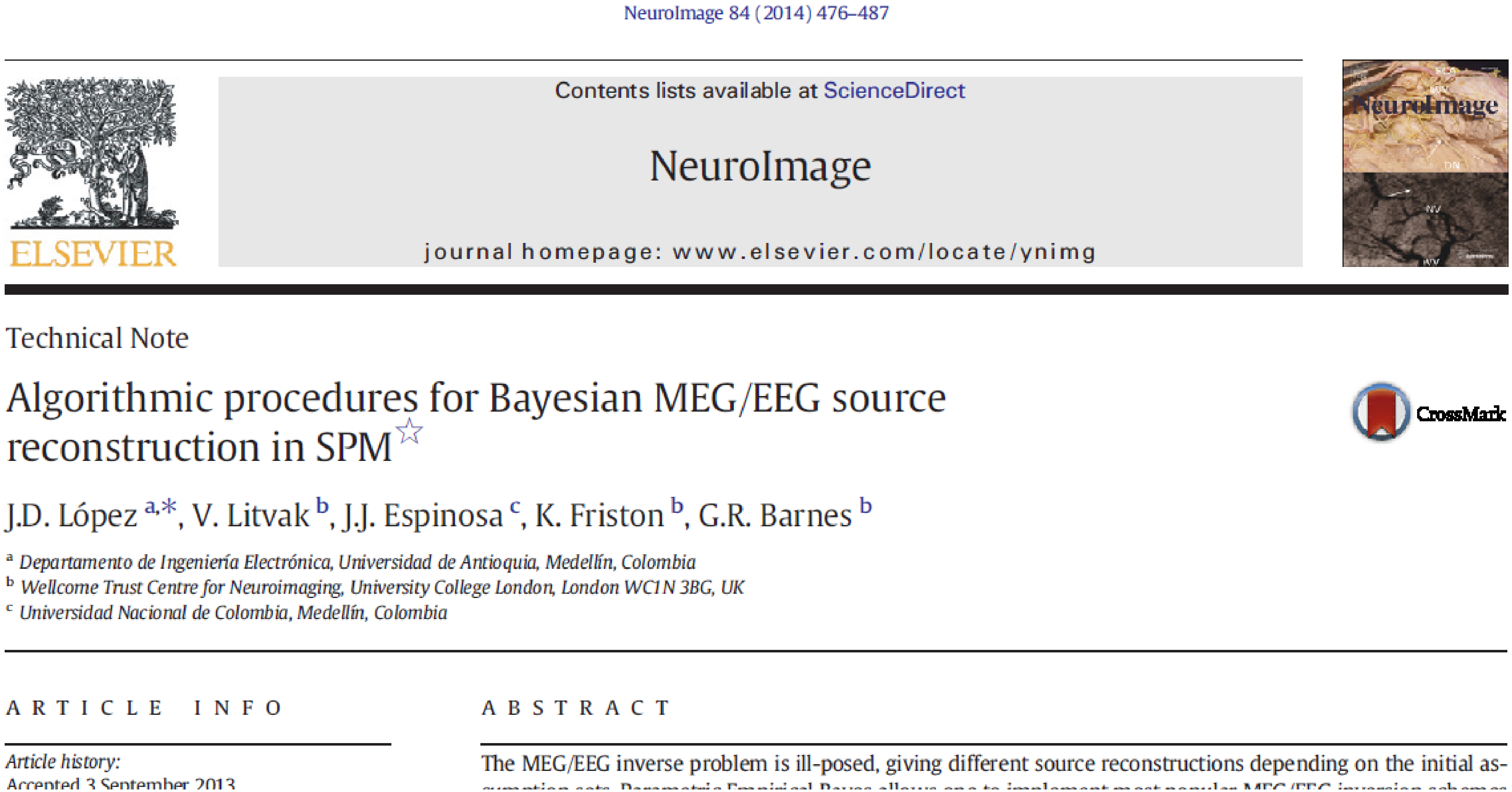
Key points :

- Prior knowledge- links to popular algorithms
- Validation of prior knowledge/ Model evidence

Conclusion

- M/EEG inverse problem can be solved... If you have some prior knowledge.
- All prior knowledge encapsulated in a source covariance matrix.
- Can test between priors (or develop new priors) within a Bayesian framework.

References



NeuroImage 84 (2014) 476–487

Contents lists available at ScienceDirect

NeuroImage

journal homepage: www.elsevier.com/locate/ynimg

Technical Note

Algorithmic procedures for Bayesian MEG/EEG source reconstruction in SPM☆

J.D. López [a],*, V. Litvak [b], J.J. Espinosa [c], K. Friston [b], G.R. Barnes [b]

[a] *Departamento de Ingeniería Electrónica, Universidad de Antioquia, Medellín, Colombia*
[b] *Wellcome Trust Centre for Neuroimaging, University College London, London WC1N 3BG, UK*
[c] *Universidad Nacional de Colombia, Medellín, Colombia*

ARTICLE INFO

Article history:
Accepted 3 September 2013

ABSTRACT

The MEG/EEG inverse problem is ill-posed, giving different source reconstructions depending on the initial as-

Thank you

- Karl Friston
- Gareth Barnes
- Vladimir Litvak
- Guillaume Flandin
- Will Penny
- Jean Daunizeau
- Christophe Phillips
- Rik Henson
- Jason Taylor
- Luzia Troebinger
- Chris Mathys
- Saskia Helbling

And all SPM developers